# FUZZY SETS AND FUZZY LOGIC -I

MONALISA SARDAR
ASST. PROFESSOR, MANKAR COLLEGE

# Introduction

- Real life problems (decision making, management, prediction etc.)often cannot be precisely defined. A huge amount of information needed to define the problems make them very complex.
- Simplifying the problem can lead to loss of information.
- The loss of information in order to make a complex problem manageable is uncertainty.
- Fuzzy logic can be used for uncertainty management in problems by resembling human reasoning.
- Humans can process imprecise information received from their sensory organs and take effective decisions.
- Fuzzy logic can solve numerical problems by using linguistic labels which are imprecise and incomplete.

# Introduction Contd.

- Boolean logic renders either a ‘TRUE’ or ‘FALSE’ assuming that a fact is either completely true or completely false.
- Fuzzy logic refers to a logic of approximation and gives the space for varying degree of truth.
- Computers can use this to interpret imprecise information such as “hot”, “tall”, “cold” etc.

# Classical sets

- Classical set is a collection of distinct objects called members or elements of the set. For example

  **HOT** = { 100, 90, 80, 70, 60} , where 100 is a member of set HOT

- Classical sets are concerned with absolute truths. An element is either a member of a set or not a member of a set.
- There is no partial membership in classical set.
- Membership function to define a classical set A :

$$\mu_A(x) = 1 \text{ , } x \in \text{A}$$
$$0, x \ \notin \text{A}$$

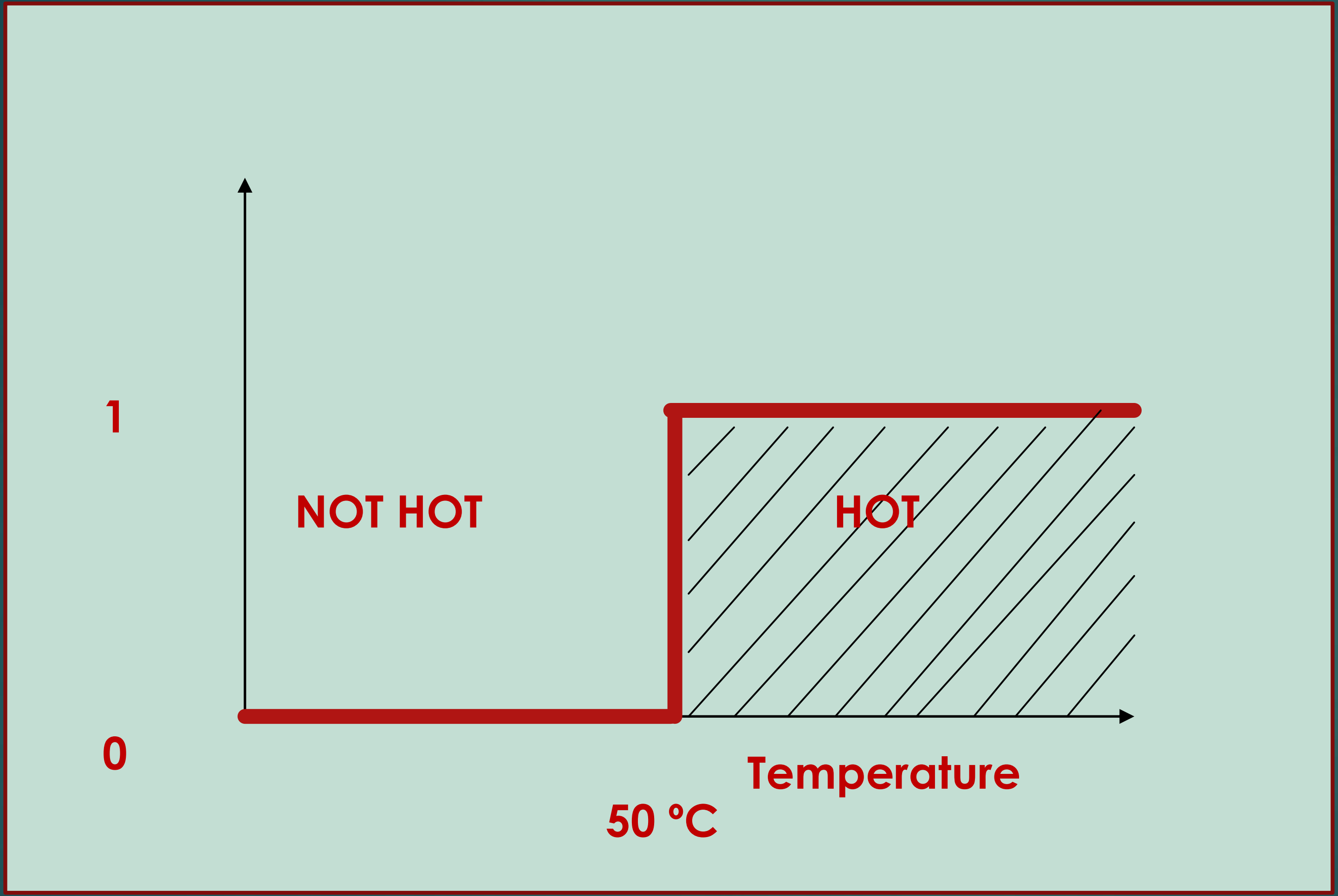
Hence, the membership function of 100 for set "HOT" is

$$\mu_{HOT}(100) = 1$$

The membership function of 200 for set "HOT" is

$$\mu_{HOT}(200) = 0$$

# Crisp/ Boolean/ Classical Logic




**If Temp >= 50 °C , it is hot (True or 1)**
**If Temp <50 ºC, it is not hot (False or 0)**

# Drawbacks of classical set

- The membership function of classical set fails to

  1. Distinguish between the members of the same set.

  2. Distinguish the differences between the members of the same set.

  For example:

  Temperature : 100 = HOT

  50.1 = HOT
  49.9 = NOT HOT

  20 = NOT HOT

There is little difference between them but they belong to different sets.
The difference between the members of the same set is huge.

# Operations on classical sets

There are two classical sets A and B.

- Complement:

$$A' = \{x | x \notin A\}$$

- Intersection:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

- Union:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

# Properties of Classical Sets

- Commutativity

$$A \cup B = B \cup A$$

$$A \cap B = B \cap \mathbf{A}$$

- Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributivity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- Idempotency

$$A \cup A = A$$

$$A \cap A = A$$

# Properties of Classical Sets Contd.

- Identity

$$A \cup X = X$$

$$A \cup \emptyset = A$$

$$A \cap X = A$$

$$A \cap \emptyset = \emptyset$$

- Transitivity

If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

# Fuzzy Sets

A fuzzy set is a set having members with varying degree of membership between 1 and 0. A fuzzy set **A~** can be represented as follows:

$$A\sim = \{ (x, \mu_A(x)) : \text{for all } x \in X\}$$

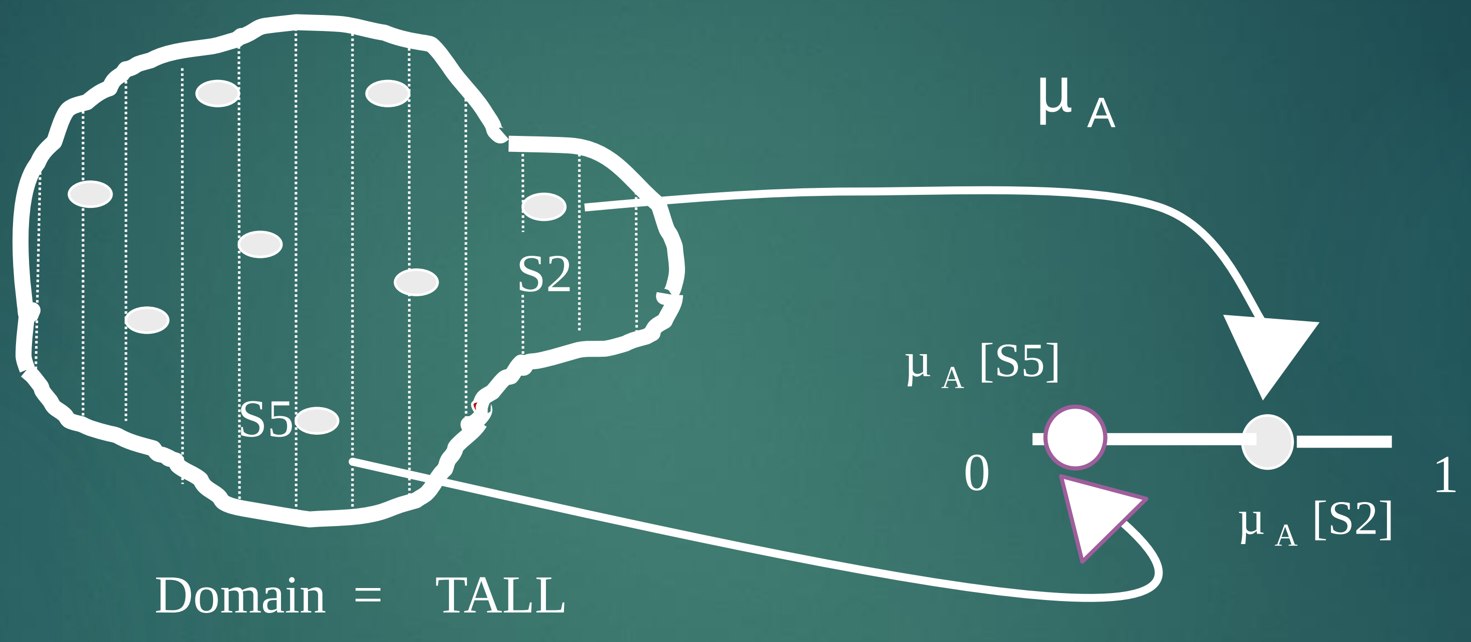
where x is the element of A~ and $\mu_A(x)$ is the membership value of x.

$\mu_A(x)$ : degree of belonging of x to A or degree of possessing some imprecise property represented by A

For example, there are 5 students in a class named S1, S2, S3, S4 and S5 and their heights are : 170 cm, 150 cm, 120 cm, 80 cm, 30 cm respectively. All of them can be a member of fuzzy set "**TALL~**" irrespective of their heights but their membership values are different from each other depending on how tall they are.

$$TALL\sim = \{ (S1,1.0), (S2,0.9), (S3,0.5), (S4,0.1), (S5,0.0001)\}$$

The membership value of S1 is more than others because he is quite tall but the membership value of S4 and S5 are less because they are not tall.




**Fuzzy sets are membership functions.**

# Fuzzy Sets Contd.

- The members of a fuzzy set have partial membership.
- The members of one fuzzy set can also be the members of other fuzzy sets with varying membership value. For example, the 5 students can also be a member of fuzzy set "**SHORT~**". But the membership value for this set is different from the membership value of **TALL~.**

  **SHORT~** = { (S1,0.0001), (S2,0.1), (S3,0.5), (S4,0.9), (S5,1.0)}

- In **SHORT~** the membership value of S1 is less than others because he is not short but the membership value of S5 is the highest because he is very short. Hence, an entity can be a member of more than one fuzzy set with different membership values.
- Fuzzy sets satisfy all the properties of classical set.

# Operations on Fuzzy Sets

There are two fuzzy sets Tall and Handsome and each have 5 members each P1 to P5.

**Tall~** = { (P1, 0.5), (P2,0.8), (P3, 0.4), (P4, 0.7), (P5, 0.1) }

**Handsome~** = { (P1, 0.9), (P2,0.2), (P3, 0.5), (P4, 0.9), (P5, 0.4) }

- UNION: C~ is the union of Tall~ and Handsome~

$$C = Tall\sim \cup Handsome\sim$$

$$\boldsymbol{\mu_C(x) = \max\{\mu_{Tall}(x), \mu_{Handsome}(x)\}\,\forall x \in Tall, x} \in \text{Handsome}$$

C~ = { (P1, 0.9), (P2,0.8), (P3, 0.5), (P4, 0.9), (P5, 0.4) }

- INTERSECTION: C~ is the intersection of Tall~ and Handsome~

$$C = Tall\sim \cap Handsome \sim$$

$$\boldsymbol{\mu_C(x) = \min\{\mu_{Tall\sim}(x), \mu_{Handsome\sim}(x)\}\,\forall x \in Tall, x} \in \text{Handsome}$$

C~ = { (P1, 0.5), (P2,0.2), (P3, 0.4), (P4, 0.7), (P5, 0.1) }

# Operations on Fuzzy Sets Contd.

- COMPLEMENT: E~ is the complement of Tall~.

$$\mu_E(x) = 1 - \mu_{Tall}(x)\ \forall\ x \in \text{Tall}$$

E~ = { (P1, 0.5), (P2,0.2), (P3, 0.6), (P4, 0.3), (P5, 0.9) }

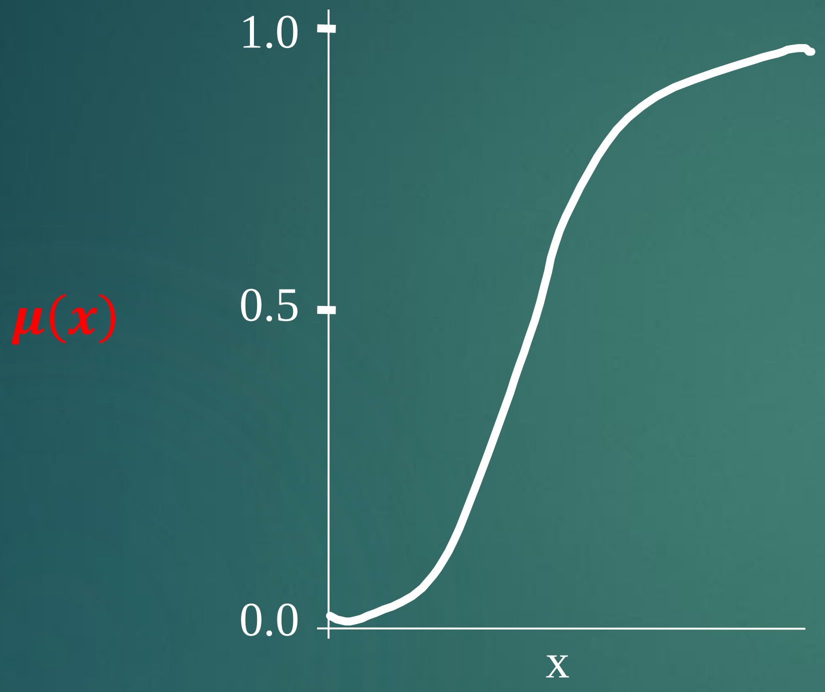
# Representation of Fuzzy Sets (Membership functions )

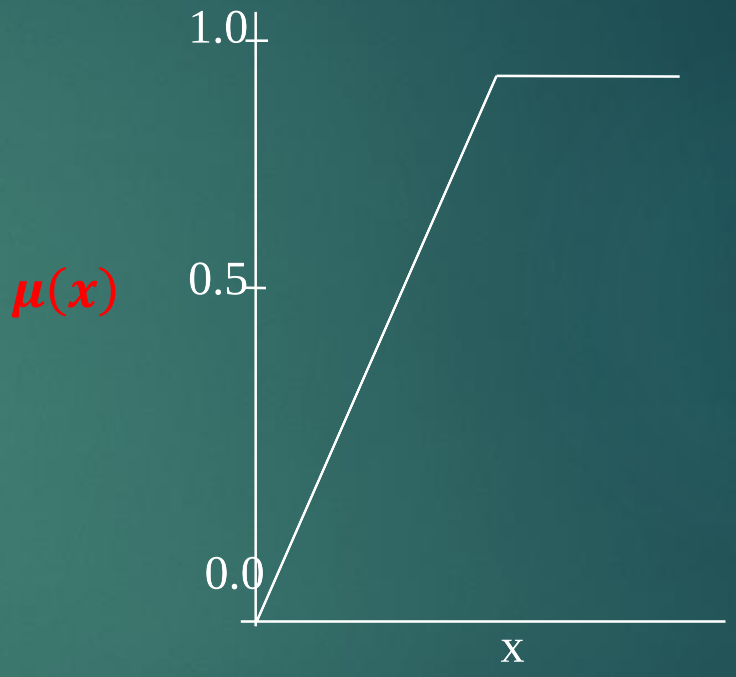
- S Representation of Fuzzy Sets (Membership functions)

- π Representation of Fuzzy Sets (Membership functions)

# S Type Membership Function



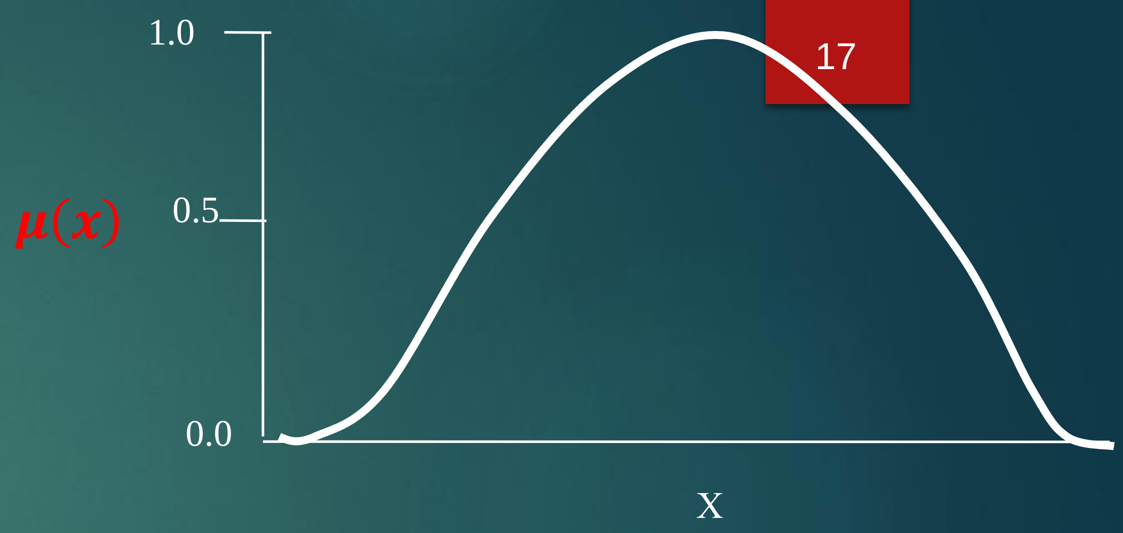


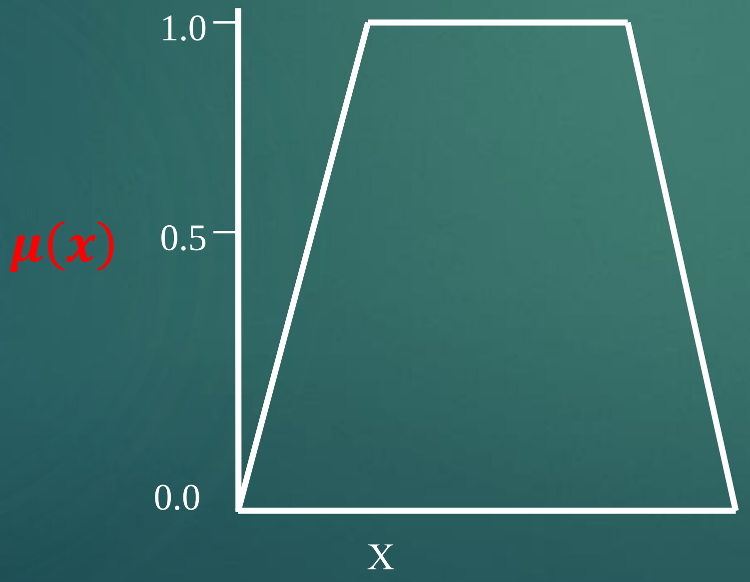



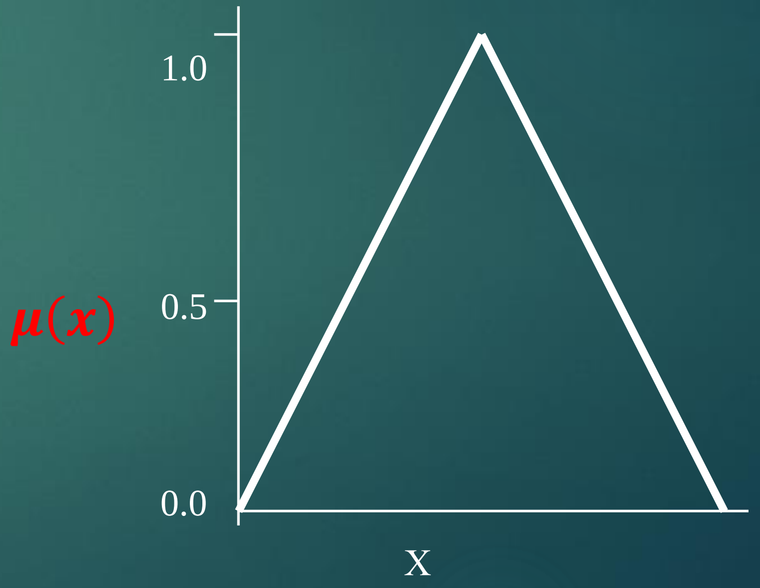

**S type functions**

# π Type membership function





# THANK YOU